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Instrumental  
Variable Price  
Elasticity Estimates  
for Water in  
Mixed-Effect  
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## **OLS and Instrumental Variable Price Elasticity Estimates for Water in Mixed-Effect Models Under Multipart Tariff Structure**

### **Abstract**

A mixed-effects residential demand model for potable water is developed using a longitudinal data set constructed for the analysis. The data set comprises of 1,065 households from the Sydney Metropolitan and Wollongong areas, covering sixteen quarters from 1990 to 1994. The purpose of developing the demand model is to use it as a base model to forecast water demand changes in response to changes in the tariff structure. The empirical results show that consumers do respond to the marginal price while faced with the multipart tariff structure. Therefore price can be considered as an influential tool in the implementation of demand management strategies. However the magnitude of price elasticity suggests that substantial increases in price would be required to influence demand.

OLS and Instrumental Variable\Maximum Likelihood estimation techniques are employed to conduct the analysis. The results support both - theory and past research, which states that IV/ML estimation technique tends to produce unbiased and consistent estimates than OLS, when price depends on quantity consumed. The Taylor/Nordin theory is also tested and the results are supportive of the theory.

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## **I. Introduction**

Over recent years with growing water demand water authorities have resorted to demand management strategies to influence water consumption, mainly by increasing the price of water to reduce consumption. Previously, the main tools used to influence water demand were water conservation programs and public education, which advocated more efficient use of water. The purpose of this paper is to develop a demand model and test the significance of marginal price in the demand equation. Do consumers react to marginal prices? If yes, water authorities can more efficiently manage their resources by implementing appropriate pricing strategies.

In NSW a trend to greater reliance on usage pricing emerged with the Hunter District Water Corporation 'pay-for-use' pricing reforms, which resulted in major cost savings. Following the Hunter Water Corporation example, Sydney Water (then the Water Board) modified their pricing structure in 1986/87. Prior to 1986/87 each household was given a water allowance determined by the property valued based rates. The higher the amount of property rate payed, the higher was the water allowance it received. Consumption above the water allowance was classified as "excess" and was charged at the usage rate. After 1986/87 Sydney Water changed the way it charged for usage. Under the new pricing policy each property was given a water allowance of 300KL per annum, irrespective of the property value. This was reduced to 250KL per annum in 1989. Finally in 1990, with the introduction of quarterly meter reading and billing, all water used was priced. This resulted in an increase in the water usage component of revenue from households from 6% in 1987/88 to 13% in 1990/91<sup>1</sup>.

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<sup>1</sup>Submission to the Government Pricing Tribunal NSW, *Pricing Reform an Investment in Sydney's Future* - September 1992, Volume 2, page 118.

In addition to Sydney Water's own initiative of movement towards a pay-for-use pricing structure, with the establishment of the Government Pricing Tribunal<sup>2</sup> (GPT) in New South

Wales there has been an increasing emphasis on efficient<sup>3</sup> pricing. The GPT<sup>4</sup> states: "the prices of water services in New South Wales need to more closely reflect the true scarcity value of the water resources." Water prices should be the true reflectors of cost and should give correct signals to the community. They also have emphasised the elimination of cross-subsidies<sup>5</sup> inherent in the price structure. At present the property component remains in the current water price schedule but there is a movement away from the property based charges<sup>6</sup> and greater reliance on usage pricing, as shown in Table 1.

Water is frequently priced using the multipart tariff structure, which is a non-uniform price schedule, and can either be two-part, increasing, or decreasing. In each case the consumer pays a fixed charge<sup>7</sup> which does not vary with the usage and a usage charge. It is the latter, that is charged differently under each case. In case of decreasing block tariff the succeeding blocks of water units are sold at lower prices. Under increasing or progressive block tariff the succeeding water units are sold at higher prices whereas in case of two-part tariff each additional water unit consumed is charged at the same price. Progressive and two-part tariffs are the most common pricing schedules because the declining block tariff

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<sup>2</sup>The Government Pricing Tribunal is a regulatory authority. It was established in 1992.

<sup>3</sup>"Efficient prices are those which lead to highest possible level of welfare, defined as the sum of consumers surplus and producers surplus," *see* Brown and Sibley ( 1986).

<sup>4</sup>1993/94 Determination of Water Board Charges.

<sup>5</sup>Cross-subsidy is said to exist when one or more group of customers are paying more than its stand alone costs ie, the true cost attributed to that group. Often it is interpreted more loosely in public discussion of water pricing.

<sup>6</sup>These will be totally eliminated by 1997.

<sup>7</sup>The fixed charge is meant to cover capital cost and overheads.

structure is advocated against, since it encourage inefficient allocation of resources and is inequitable. It is inefficient because it encourages waste since succeeding water units can be purchased at lower price and is inequitable because low income earners tend to use less water than high income earners with huge gardens.

**Rate Schedule Used in The Analysis  
Under Multipart Tariff Structure  
Table 1**

<b>Year</b>	<b>Water</b>	<b>Sewer</b>	<b>Drain</b>	<b>Water</b>	<b>Water &amp; Sewer</b>	<b>Water, Sewer &amp; Drain</b>	
<b>Fixed Charges</b>							
<b>1990/91</b>							
Base Charges	\$23.250	\$57.500	\$3.000				
Pensioner Rebate				\$20.000	\$46.000	\$47.000	
Property Rates*	0.043	0.109	0.017				
Maximum	39.000	122.700	13.870				
<b>1991/92</b>							
Base Charges	24.300	60.480	3.150				
Pensioner Rebate				21.040	48.390	49.440	
Property Rates*	0.045	0.115	0.017				
Maximum	40.950	128.830	14.560				
<b>1992/93</b>							
Base Charges	24.250	63.200	3.290				
Pensioner Rebate				22.840	51.410	52.510	
Property Rates*	0.045	0.115	0.017				
Maximum	40.830	128.640	14.880				
<b>1993/94</b>							
Base Charges	20.000	63.000	3.750				
Pensioner Rebate				20.000	51.500	53.370	
Property Rates*	0.045	0.115	0.017				
Maximum	37.000	128.000	14.880				
		<b>Usage Charges</b>					
<b>Year</b>	<b>Quarter 1</b>	<b>Quarter 2</b>	<b>Quarter 3</b>	<b>Quarter 4</b>			
<b>Litres/day</b>	<b>Jul - Sep</b>	<b>Oct - Dec</b>	<b>Jan - Mar</b>	<b>Apr - Jun</b>			
<b>1990/91</b>	<i>Increasing Block Tariff Structure</i>						
0 - 600	\$0.15	\$0.15	\$0.15	\$0.15			
601 - 822	0.27	0.27	0.27	0.27			
822 - 30,000	0.55	0.55	0.55	0.55			
above 30,000	0.55	0.55	0.55	0.55			
<b>1991/92</b>							
0 - 600	0.15	0.15	0.16	0.16			
601 - 822	0.27	0.27	0.29	0.29			
822 - 30,000	0.55	0.55	0.59	0.59			
above 30,000	0.55	0.55	0.64	0.64			
<b>1992/93</b>							
0 - 600	0.16	0.21	0.21	0.21			
601 - 822	0.29	0.30	0.30	0.30			
822 - 30,000	0.59	0.59	0.59	0.59			
above 30,000	0.64	0.64	0.64	0.64			
<b>1993/94</b>							
0 - 600	0.21	0.21	0.65	0.65			
601 - 822	0.30	0.30	0.65	0.65			
822 - 30,000	0.59	0.59	0.65	0.65			
above 30,000	0.64	0.64	0.65	0.65			

Note:

\*c/\$ for land value above \$33,000.

Over the time of the study, the price schedule took the form of either increasing or two-part tariff comprising two components as mentioned before i.e., fixed charge and usage charge. The fixed charge usually just comprises the base charge<sup>8</sup> which is fixed but in case of Sydney Water the fixed charge is made of two components the base charge which is fixed and the property rate which varies with the property value. The property component is gradually being reduced and will be totally eliminated by next year. The usage charge for the first fourteen quarters of the study period consists of an increasing block tariff structure which is common among the developing countries,<sup>9</sup> where the income redistribution objective is given a priority and the water authority undertakes the workings of the social security agencies. In case of developed nations like Australia, the main objective of the water authority is to cover the total cost in the most efficient manner and the "closest practical approach to efficient pricing is two-part tariff"<sup>10</sup> (the fixed part of the tariff covers the fixed costs and the operating costs are covered by the usage revenue). Following the economics principle of efficient pricing Sydney Water converted to two-part tariff in 1993/94.

This paper proceeds as follows: Section II discusses various past approaches used to address similar demand modelling issues. Section III presents details about the data set used and Section IV specifies the methodology adopted by the paper and the demand function used in the analysis. Section V discusses the results derived and finally Section VI provides concluding comments.

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<sup>8</sup>The base charge covers the costs that is above marginal cost. This is appropriate in case of natural monopolies, like water which face a declining average cost, mainly attributable to economies of scale. In such a situation if the organisation resorts to marginal cost pricing it will not be able to break-even because when average cost is falling marginal cost is below average cost. Therefore it has to resort to above marginal cost pricing to cover additional costs in the form of base or fixed charge.

<sup>9</sup>Pricing of Water Services, OECD 1987.

<sup>10</sup>Pricing of Water Services, OECD 1987.

## II. Past Approaches to Modelling

Prior to the 70's most of the studies that estimated demand under a multipart tariff structure used average price (AP) as the only explanatory price variable [see Wong 1972, Young 1973, Gottlieb 1963 & Foster and Beattie 1979]. Most of these studies calculated the AP per household as Total Bill/Total Consumption and argued that the consumers respond to the total bill rather than the marginal price because they have more accurate information about their bills rather than the block nature of pricing. Thus earlier studies did not account for any intramarginal effects caused by the block structure pricing.

The concept of second price<sup>11</sup> along with the marginal price (MP) was introduced by Taylor [1975]. He suggested that a single price variable AP or MP is not sufficient - in fact the entire demand schedule should be represented in the demand function, stressing the importance of capturing the budget constraint facing the consumers. His theory was further developed by Nordin [1976] who introduced a difference variable often referred to as the rate structure premium (RSP)<sup>12</sup>. The RSP is called the difference variable because it is the difference between the total bill less what the bill would have been if the water quantity was consumed at the marginal price. This is defined as follows:

$$AP = \text{Bill}/Q = MP + RSP/Q^{13} \quad (1)$$

$$\Rightarrow RSP = (AP - MP)*Q \quad (2)$$

The RSP<sup>14</sup> should be able to capture the income effects of changes in the intramarginal prices, the fixed price and the quantity breakpoints. Nordin also

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<sup>11</sup>Billing and Agthe [1980].

<sup>12</sup>He emphasised that the consumers react not only to marginal prices but also to the changes in the consumer surplus as a result of moving from one block to the other and that these intramarginal effects should be included in the demand equation [Chicoine, Deller and Ramamurthy 1986]. The difference variable in terms of consumer surplus is described as the difference in the consumer surplus under marginal pricing and the consumer surplus that is actually experienced by a typical consumer. In case of increasing (decreasing) block tariff the consumer surplus is larger (smaller) than if the units were purchased at marginal price.

<sup>13</sup>Jeong-Shik Shin [1985].

hypothesised that the coefficients of RSP and income variables should be equal in magnitude but opposite in sign because each measures a pure income effect therefore their coefficients in a linear demand equation should be equal. The expected sign of income is positive but the derivative of water use with respect to difference is negative because increasing the intramarginal rates increases the difference and the implicit tax which reduces water use.

The Taylor/Nordin theory<sup>15</sup> has been tested in several studies in both the electricity and water industries. The results of Billing and Agthe [1980] support the theory to some extent since they found that the rate structure premium (RSP) and income co-efficients have opposite signs but significantly different magnitudes. They attribute the limitation to the use of aggregate rather than individual data. In case of water, Jones and Morris [1984] tested the theory but their results were not supportive. The coefficients on these variables were also found to be of unequal magnitude in case of an electricity industry study conducted by Taylor and Blattenberger [1977].

In response to the failure of past empirical research to validate the Taylor/Nordin theory, Jeong-Shik Shin [1985] argued that the cause might be price illusion or incomplete information concerning the full budget constraint. In such a situation to capture the pure income effect Shin introduces a perceived price variable called the price perception variable<sup>16</sup> in his "Price Perception Model", in addition to the marginal price. The price perception variable is a function of average price, marginal price and a price perception parameter. His paper examines the price information problem that the consumers face. The empirical results of the Shin price perception model show that consumers respond to average price rather than marginal price when faced with the decreasing block rate structure. His model was tested by Nieswiadomy and

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<sup>14</sup>In multipart tariff structure it is required to capture the correct intercept.

<sup>15</sup>Which states that the rate structure premium and the income co-efficients should be opposite in signs but equal in magnitude.

<sup>16</sup>Jeong-Shik Shin [1985]

Molina [1991] for increasing block tariff structure, who concluded that water consumers react more to marginal price than average price. But whether consumers respond to average or marginal price is still a debatable issue because William [1985] using similar data to Shin [1985] found that consumers respond to marginal price.

This study does not use the price perception model, but develops a demand model which includes the RSP, the marginal price and income along with other explanatory variables.

The RSP is illustrated on the following page, for the case of an increasing block tariff. The increasing steps in part (a) of the Figure are the result of an increasing block tariff structure. Part (b) of the Figure also illustrates how the slopes of the price segments increase as consumption increases.



The y-axis measures the income spent on a composite of all other goods excluding water and the x-axis represents water consumption. The budget constraint faced by the consumer is the piecewise linear segment ABCD. Suppose the consumer consumes  $Q_0$  units of water where the indifference curve  $I_0$  is tangent to the linearised budget line YG at E. Income is OF and the fixed or access charge is FA. In such a situation, if only average price (indicated by the slope of the line FH passing through point E) is used in the analysis, the intercept will be F, which exaggerates the price. On the other hand, if only the marginal price is used, the appropriate intercept is Y, which is the marginal price line extended to the vertical axis when  $Q_0$  is consumed. In this case, the correction required to obtain an accurate intercept is the distance FY, which is the rate structure premium. The use of the virtual income OY (equal to Income-RSP) is consistent with the theory of consumer choice if a piecewise linear budget constraint is linearised to give a single marginal price (P2). The virtual income intercept supports the chosen consumption as a tangency. In the case of an increasing block tariff, the consumer can over or underestimate the true marginal price by using the average price, with the extent of the error depending on the magnitude of the access charge. A small access charge leads to underestimation of the true marginal price and a large access charge leads to overestimation of the true marginal price (Nieswiadomy and Molina 1991). The use of the average price as the only explanatory price variable in the analysis in the case of the non-uniform pricing structure leads to exaggerated price elasticities. Therefore, in the case of a block tariff structure, to capture the correct intercept, it is essential to include the rate structure premium in addition to the marginal price and the actual income, otherwise the parameters estimated are biased.

### III. Data

The data used in the analysis is a cross-section time-series longitudinal or panel data. The cross-section data is based on the survey conducted by the Government Pricing Tribunal. The total survey sample comprised of 400 collectors district, of which 352 were picked from Sydney Water supply area. In each collectors district 5 households were randomly chosen and interviewed. The GPT survey covers the Sydney Local Government Areas (LGA) including Penrith, Campbelltown, Camden, Hawkesbury, Lake Macquarie, Blue Mountains, Wollongong, Kiama and Shellharbour. The main objective of the survey was to have representative samples of households in the specified catchment areas. The cross-section<sup>17</sup> data for 1992 provides information on demographics, income and property values.<sup>18</sup> The survey collected the information on separate dwellings<sup>19</sup> and units but only the information on 1,065 separate dwellings served by Sydney Water is used in the present analysis because the quarterly data on water usage<sup>20</sup> (in kilolitres) was only available for these household.

A panel data set was constructed in order to improve the precision of demand estimates. The GPT survey data is amalgamated with quarterly data on water usage (in kilolitres), weather<sup>21</sup> and rate structure. The rate structure or the price

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<sup>17</sup>The survey data (cross-section) was only available for 1992. Since similar information was not available for other years it was assumed that the demographics, property value and income variable remained constant for the last two years (1990 and 1991) and for the next two years (1993 and 1994). It is very unlikely that the number of bathrooms, toilets or property value would change much though the number of people per household and the income variable might vary, but these are also assumed to be constant over time in real terms. The income and property values are deflated and inflated to specify them in real \$1994.

<sup>18</sup>Additional information that the survey collected per household was the type of water efficient appliance installed, (eg dual flush toilets), blocksize, number of part-time or full-time workers, number of people less than 15 year. These variables are not used in the demand function because they did not improve the model.

<sup>19</sup>Separate dwelling comprises of houses, villas, townhouses and duplexes.

<sup>20</sup>The panel data obtained for Sydney Water for 1,065 households from 1990-1994 for sixteen quarters..

<sup>21</sup>The quarterly (1990-1994) weather data for rainfall and temperature is obtained from the Bureau of Meteorology.

schedule (for sixteen quarters) used in the analysis is non-uniform. The households face an increasing block tariff for the first fourteen quarters and a two-part tariff for the remaining two quarters. All values are specified in \$1994. Summary statistics are given in Table 2.

The RSP captures the correct intercept in case of a block rate structure and is calculated as follows:

$$\text{RSP} = (\text{Bill} - \text{MP} * \text{Q})^{22} \quad (3)$$

The consumer bill is the total bill which includes;

$$\text{Bill} = \text{SAC} + \text{PT} + \text{UC} \quad (4)$$

where

SAC = Service Availability Charge

PT = Property Tax and

UC = Usage Charge

The bill is calculated separately for pensioners and non-pensioners because pensioner rebates had to be taken into account while computing the costs of water consumption.

The other variable in equation (3) is MP or the price consumers would pay with each additional use, where the use is specified in each block<sup>23</sup> to which the MP is attributed.

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<sup>22</sup> Derived from equation (1) in section I.

<sup>23</sup> The rate schedule is given in Table 1.

## Summary Statistics

Table 2

Variable	Units	Mean	Standard Deviation
No of Observations -----> 17,024			
Water Consumption	Kilolitres	73.45	124.83
<i>Price</i>			
Marginal Price	\$ 1994	0.37	0.19
Average Price	\$ 1994	3.74	5.74
Rate Structure Premium	\$ 1994	121.41	44.15
Base	\$ 1994	131.33	43.25
Bill	\$ 1994	155.67	78.58
<i>Income</i>			
Income	\$ 1994	10,041.76	6,574.76
Property Value	\$ 1994	260,302.27	173,115.34
<i>Demographics</i>			
Bedrooms		3.22	0.84
Toilets		1.74	0.75
Household Size		3.25	1.46
<i>Weather</i>			
Rainfall	Millimetres	87.81	50.55
Temperature	Centigrades	21.25	3.77

#### **IV. Methodology and Estimation Technique**

The methodology employed in this paper is the one developed by Nordin which is a modification of Taylor's [1975] theory. To apply the theory, a mixed-effects demand model (equation 5) is developed to conduct the analysis on the panel data set constructed.

$$Q_{it} = \beta X_{it} + \gamma R_i + \omega_{it} \quad (5)$$

$$\omega_{it} = v_i + \varepsilon_t + \mu_{it} \quad (6)$$

$$\varepsilon_t = \rho \varepsilon_{t-1} + \lambda_t \quad (7)$$

$$v_i \sim N(0, \sigma_v^2)$$

$$\lambda_t \sim N(0, \sigma_\lambda^2)$$

$$u_{it} \sim N(0, \sigma_u^2)$$

where:

$i$  is a cross-sectional index representing each household,  $t$  is quarterly time series index and  $\rho$  is the autocorrelation estimate.

The mixed-effects model takes into account both fixed and random effects.  $\beta$  and  $\omega_{it}$  are vectors of fixed and random effects, respectively. The error term  $\omega_{it}$  takes into consideration the cross-section disturbance term ( $v_i$ ) which measures the shock across households, the time-series disturbance term ( $\varepsilon_t$ ) with the first order autoregressive error structure ( $\varepsilon_{t-1}$ ) and a combined error component ( $u_{it}$ ). The assumptions imply that the cross-sectional errors are uncorrelated over time but the time-series disturbances are correlated over time.

The maximum likelihood (ML) estimation technique is employed to estimate the demand model. The Newton Raphson iterative procedure is used to maximise the likelihood function with respect to the parameters specified in the model and tries to locate an optimum subject to the demand model specified.

Panel data sets have advantage over either time-series or cross-section data sets because they increase the number of data points and lead to more degrees of freedom. They also add a new dimension to the problems of model specifications. The additional problem is the structure of the error term which

gets more complex because it includes both time-series and cross-sectional related disturbances. Thus a complex stochastic structure is specified in equation (6) and (7).

An additional model specification problem in estimating the demand function is the endogenous price variables. Under an increasing block tariff structure the non-linear budget constraints<sup>24</sup> cause the price variables to be endogenous. The price and rate structure premium determined by the quantity demanded<sup>25</sup> leads to a "reverse causality"<sup>26</sup> problem which has to be corrected for otherwise the demand estimates are biased, inefficient and inconsistent. The paper employs the Instrumental Variable (IV) estimation technique which addresses the problem of endogenous price variables (MP) and (RSP) which are correlated to the error term ( $\omega$ ) on the right hand side of the regression equation (8).

$$X = f(\text{MP}, \text{RSP}, Z) + \omega \quad (8)$$

Z are the other explanatory variables including income. In this situation the use of OLS estimation generates biased, inefficient and inconsistent estimates.

In order to address this problem an instrument variable is required, a variable that is highly correlated with the price variable but uncorrelated with the error term,  $\omega$ . Once such a variable is found, OLS or ML estimation techniques can be used.

Though several instrumental variable techniques<sup>27</sup> have been suggested. The two stage instrumental variable estimation technique used in this analysis is

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<sup>24</sup>Which leads to the problem of the "Kinked Budget Constraints", it is assumed for the purpose of this analysis that none of the observations lie at the kink - thus everyone is assumed to be located at one of the price segments.

<sup>25</sup> $P=f(Q) \Rightarrow Q=f(P)$

<sup>26</sup>Robert Moffit[1990].

<sup>27</sup>These are suggested by Wilder and Willenborg [1975], McFadden, Puig, and Kirscher [1977], Henson [1984] in the Electricity Industry and Billing and Agthe [1980], Jones and Morris [1984], Chiocone, Deller Rammamurthy [1986] and Nieswiadomy and Molina [1991] in the Water Industry.

analogous to the one used by Hausman and Wise [1976] and Rosen [1976] in the labour supply cases, by Hausman, Kunnican and McFadden [1979] and Terza [1986] in the electricity industry and by Nieswiadomy and Molina [1989] in the water industry demand models. The stages involved in the development of the price instruments are given in detail in the following section.

In the first stage, the water demand is estimated on the set of actual marginal prices that are faced by each household at the three predetermined consumption levels (600, 822 and 823 ) chosen to capture the budget set, as specified in equation (9);

$$Q_{it}^* = f(P_{it}, B_{it}, X_{it}) \quad (9)$$

Where  $P_{it}$  is a vector of prices corresponding to the exogenous quantities.

$B_{it}$  is the base charge

$X_{it}$  are exogenous variable<sup>28</sup> used in the analysis

The predicted water consumption estimated in stage 1 is used to calculate the predicted marginal price and the predicted rate structure premium, specified in equation (10) and (11), respectively.

$$MP_{IV} = f(Q_{it}^*) \quad (10)$$

$$RSP_{IV} = f(Q_{it}^*) \quad (11)$$

In stage 2, these instrument price variables are used as independent variables in the demand model (equation 12) to estimate water consumption.

The mixed-effects demand function used in the analysis is :

$$\log Q_{it} = a + b \log RSP_{IVit} + c \log MP_{IVit} + d \log T_t + e \log R_{t-1} + f \log Y_{it} + g \log PV_{it} + h P_t + i X_{it} + \epsilon_{it} \quad (12)$$

Where:  $i=1..1,065$  (households) and  $t=1..16$  ( quarters)

$Q_{it}$  denotes the quantity of water demanded by the  $i$ th household in  $t$ th quarter.

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<sup>28</sup>Household size, temperature, rainfall, property value and income are used.

$RSP_{IVit}$	is the difference variable calculated for each household for each quarter.
$MP_{IVit}$	is the predicted marginal price faced by $i$ th household in $t$ th quarter.
$T_t$	is the average temperature variable in $t$ th quarter.
$R_{t-1}$	lagged rainfall (lagged by one quarter)
$Y_{it}$	income of the $i$ th household in $t$ th quarter (using the midpoints of the relevant ranges specified in the series)
$PV_{it}$	market property value of $i$ th household in $t$ th quarter. This is the expected price of the property as perceived by each household (midpoints of the property value range are used in the calculation)
$P_t$	a dummy for peak and off peak ie, Dt=1 for summer quarters - peak Dt=0 for winter quarters - off peak
$X_{it}$	demographic variables used in the analysis comprising of household size, bedrooms and the number of bathrooms\ toilets plus the household garden condition
$\omega_{it}$	refer to equation (6) and (7)

The model adopted is a log-linear form so that the coefficients of price and income are specified as elasticities. The log-linear model also provides a better fit (higher  $R^2$ ) than the linear one.

## **V. Results**

The results of the analysis are presented in table 3. The water demand estimates are calculated using OLS and IV\ML estimation techniques. The OLS estimates are given for the purpose of comparison. The IV\ML techniques are used to deal with the model specification problems discussed in section IV.

The results presented in table 3 are interesting from many angles. First, the wrong sign of the price coefficient: 0.25 under OLS depicts the inherent bias. The bias in OLS can get extremely large and in certain cases can reverse the expected sign of price and income elasticities [Dubin 1982; Medgel 1987; Moffit and Nicholus 1982], in case of this study only the price variable has a wrong sign. Similar results are reported using an increasing block tariff structure by Nieswiadomy and Molina [1989] - who conclude that the bias in the price coefficient is positive in case of increasing block tariffs. The results of this study also show the inherent positive bias. The signs of other coefficients under OLS are as expected except for temperature but the t-ratios are slightly exaggerated because of a bias in the estimated variances. In addition to this the presence of autoregressive error structure ( $dw=1.49$ ) can also cause misleading parameter estimates and levels of significance.

**Water Demand Estimates  
Under Multipart Tariff  
Table 3**

Variables	OLS	IV 2nd Stage <i>IV/ML</i>	IV 1st Stage
Intercept	5.07 <i>*36.25</i>	2.57 <i>*5.79</i>	1.64 <i>1.18</i>
MP	0.25 <i>*16.15</i>	-0.21 <i>*-4.11</i>	----
RSP	-0.19 <i>*-50.97</i>	-0.03 <i>-2.59</i>	----
HH Size	0.05 <i>*14.56</i>	0.17 <i>*11.78</i>	0.17 <i>*20.28</i>
Toilets	0.03 <i>*3.88</i>	0.08 <i>*5.24</i>	
Bedrooms	0.01 <i>1.15</i>	0.04 <i>*2.97</i>	
Temp	-0.63 <i>*-16.98</i>	0.04 <i>0.23</i>	0.03 <i>0.71</i>
Peak	0.32 <i>*30.05</i>	0.26 <i>*5.09</i>	0.15 <i>0.68</i>
Lag Rainfall	-0.05 <i>*-6.96</i>	-0.13 <i>*-3.81</i>	0.00 <i>-1.16</i>
Garden	0.01 <i>0.77</i>	<i>0.05</i> <i>2.43</i>	
Prop Val	0.08 <i>*9.69</i>	0.01 <i>0.84</i>	0.08 <i>*6.83</i>
Income	0.05 <i>*9.95</i>	0.07 <i>*10.64</i>	0.07 <i>*7.39</i>
Base			-0.06 <i>*-4.28</i>
Block 1			-1.21 <i>-0.91</i>
Block 2			1.59 <i>0.84</i>
Block 3			0.26 <i>0.09</i>
R-sq	0.58	0.62	
STDEV	0.5041	0.588	0.5927
<i>Rho</i>	0.255	0.0105	0.044
DW(apprpx)	1.49	1.98	1.91
Null Model LRT		4651	3961

Notes:-

The figures in italics are T-ratios, '\*' implies significance at 1% level.

In contrast to the OLS estimates the results under IV\ML estimations are supportive of past theory. The coefficient of price elasticity is -0.21, which is inelastic with negative sign as expected. Since water is a necessity the degree of

responsiveness of water with respect to price is less than unity. This is also consistent with the income elasticity of 0.07, which lies within the range of previous studies given in Table 4.

The signs of the other variables are as expected. The t-ratios of all other variables are significant except for temperature and property value. The t-ratios in Table 3 for the second stage have been adjusted for the inherent bias in the coefficient's standard errors<sup>29</sup>. The peak dummy is significant implying that during summer the demand is higher. The rain variable is lagged by one quarter assuming that the heavy rainfall in last quarter will reduce consumption in the next quarter. The demographic variables ie, household size, bedrooms and toilets all have the expected positive signs. The garden condition variable is significant which implies that households with well maintained gardens tend to consume more.

**Elasticity Estimates of Various Studies**  
**Table 4**

<b>Studies</b>	<b>Marginal Price</b>	<b>Rate Structure Premium</b>	<b>Income</b>
This study <i>Panel Data</i>	-0.21	-0.03	0.07
Billing and Agthe (1980) <i>Time Series</i>	-0.27	-0.12	1.68
Jones and Morris (1984) <i>Cross Section</i>	-0.18	-0.24	0.40
Chicoine, Deller & Ramamurthy (1986) <i>Cross Section Rural</i>	-0.22		0.01
Nieswisdomy and Molina (1989) <i>Time Series</i>	-0.86		0.14

<sup>29</sup> These are multiplied by the ratio of estimated standard deviations (see Maddala, page 373-377).

The property value variable included is the market value of the property which has a positive sign as expected but is insignificant. This implies that more expensive properties tend to use more water.

Another important point to note is that the income and RSP coefficients are opposite in sign as expected but are not equal in magnitude. The results shown in table 4 support the former part of the hypothesis.<sup>30</sup> The latter part of the hypothesis which specifies that the magnitude of the income and RSP coefficient should be the same applies to the derivatives and not to elasticities. Since the elasticities are specified in Table 3 the coefficients of income and price variables are derived<sup>31</sup> which are 0.0005 and -0.018, respectively. The co-efficients are significantly different in magnitude which supports the previous empirical research suggesting some price illusion.

The goodness of fit ( $R^2$ ) for the IV/ML second stage is 0.62 which is good given the amount of time-series data available and is better than the  $R^2$  under OLS model. The model has also been corrected for autocorrelation with the given estimated value of 0.0105 of  $\rho$  and DW of 1.98. The null model likelihood test ratio (LTR) is 4651, which shows a significant difference between the null model and the likelihood model. The null model is the standard linear model with only fixed effects<sup>32</sup> and the residual. This implies that the first order autoregressive covariance matrix is preferred to the diagonal one of the OLS null model.

<sup>30</sup>Billing and Agthe [1980] and Jones and Morris [1984].

<sup>31</sup>The income ( $\epsilon_y$ ) and price ( $\epsilon_{rsp}$ ) elasticity values are taken from table 3 whereas the mean values are taken from table 2.

$$\begin{aligned} \epsilon_y &= \partial Q / \partial Y * Y_{Mean} / Q_{Mean} \\ \Rightarrow \partial Q / \partial Y &= .0695 * (73.45 / 10,041.76) \\ &= 0.0005 \end{aligned}$$

and

$$\begin{aligned} \epsilon_{rsp} &= \partial Q / \partial RSP * RSP_{Mean} / Q_{Mean} \\ \Rightarrow \partial Q / \partial RSP &= - 0.0297 * (73.45 / 121.41) \\ &= - 0.018 \end{aligned}$$

<sup>32</sup>Including intercept and all independent variables.

## **VI. Conclusion**

This paper has estimated a water demand model under a multipart tariff structure consisting of an increasing or two-part tariff structure depending on the date. The purpose of developing the demand model is to use it as a base model to forecast water demand<sup>33</sup> changes in response to changes in the tariff structure. Two estimation techniques are used which are OLS and IV\ML. The OLS estimates are likely to be biased and inconsistent with the theory of structural demand models where equations are specified as supply-demand models. The results are also supportive of both the theory and past research which states that IV/ML estimation technique should be used in case of mixed-effects simultaneous equation models where the error term is of a complex stochastic form with an inherent autoregressive error structure and is correlated with the price variable.

The estimated parameters of the model have expected signs and nearly all are significant at 1% level of significance. The price elasticity of income is less than one, which is supportive of past research<sup>34</sup> since water is a necessity, this is supported by the highly inelastic income elasticity.

The empirical results show that consumers do respond to the marginal price while faced with the multipart tariff structure. Therefore price can be considered as an influential tool in the implementation of demand management strategies. However the magnitude of price elasticity suggests that substantial increases in price would be required to influence demand.

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<sup>33</sup>Assuming that the independent variables used in the analysis follow the same pattern of growth.

<sup>34</sup>Table 4 gives a comparative analysis of income and price elasticities. The price elasticity is inelastic and so is income except in case of Billing and Agthe [1980] which show an elastic income elasticity.

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